CS 6212 – Final

Name:

Score:

60 points. 120 minutes

Q 1 – 4 (5 points each): ***[Only write answers, NO explanations]***

Questions 1, 2: What is the time complexity of these algorithms/functions, in terms of n? ***[Sum += y is a short form notation for Sum = Sum + y.]***

`for (int i = 1 to n) {`
`  for (int j = i to n) {`
`  for (int k = j to n) {`
`  Sum += a[i]*b[j]*c[k]`
`  }`
`  If (j % i == 2) {`
`  j = n`
`  }`
`  }`
`}`
	`j = 1`
	`while (j < n) {`
	`  k = j`
	`  while (k < n) {`
	`  Sum += a[k]*b[k]`
	`  k = k * k`
	`  }`
	`  if (k > 3 * j) {`
	`  j = 2 * j`
	`  } else {`
	`  j++`
	`  }`
	`}`

3. (5 points) Solve recurrence relation: T(n) = 5 T(n/3) + n log n.

4. (5 points) Solve recurrence relation: T(n) = T(n-42) + n/4.

5. (10 points) ***Snow White and 7 Conference Rooms:*** You are the chief algorithm officer for Snow White Inc. Because of your algorithmic abilities, Snow White Inc. is very successful and has hundreds of thousands of investors. You are planning the next investor conference. You want to make sure that all investors who know each other attend the meeting in different rooms. You are trying to decide if the 7 conference rooms in the Snow White Inc. head quarters will be sufficient or not. Prove that this decision problem is NP-complete by using one of the known NP-complete problems (CLIQUE, 3-SAT, Hamiltonian Path, Hamiltonian Cycle, Coloring, Independent Set, etc.) (As a free public service, explanations of all these problems are provided on the last page.)

6. (10 points) You are given two sorted arrays *A* and *B* of size *n* and *m* respectively. Find the median of *A U B*. The overall run time complexity of the algorithm should be *O(log (m+n))*.

7. (10 points) Greedy algorithm for TSP can be defined as "select the cheapest edge out from current node to an unvisited node". Give an example of a graph where a greedy algorithm for finding a TSP does not yield optimal result.

8. (10 points) **SAT in polynomial time! Finally!** You are given the Satisfiability problem, with a slight twist that instead of 3, there are only two literals in each disjunctive clause. For example: "(a OR b) and (a' OR c)". You need to solve this problem in polynomial time and analyze its time complexity. Provide the best algorithm possible. What is the time complexity of the proposed algorithm?

This hint may help: (a OR b) and (a' OR c) is equivalent to (b OR c)

9. (4 Bonus points) Explain the Project P4 belonging to a DIFFERENT TEAM in 4 sentences.

Known NP Complete Problem Definitions

CLIQUE: The clique problem is the computational problem of finding cliques (subsets of vertices, all adjacent to each other, also called complete subgraphs) in a graph.

3-SAT: In logic and computer science, the Boolean satisfiability problem (sometimes called propositional satisfiability problem and abbreviated SATISFIABILITY, SAT or B-SAT) is the problem of determining if there exists an interpretation that satisfies a given Boolean formula.

Hamiltonian Path, Hamiltonian Cycle: In the mathematical field of graph theory the Hamiltonian path problem and the Hamiltonian cycle problem are problems of determining whether a Hamiltonian path (a path in an undirected or directed graph that visits each vertex exactly once) or a Hamiltonian cycle exists in a given graph (whether directed or undirected). Both problems are NP-complete.

Coloring: Graph coloring is a special case of graph labeling; it is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color; this is called a vertex coloring.

Independent Set: In graph theory, an independent set is a set of vertices in a graph, no two of which are adjacent. The independent set problem is correspondingly the problem of deciding whether or not the given graph contains an independent set of the given size.